Cheat Sheet of Mathemtical Notation and Terminology

Logic and Sets

Notation	Terminology	Explanation and Examples
$a := b$	*defined by*	The object a on the side of the colon is defined by b.
		Examples: $x := 5$ means that x is defined to be 5, or $f(x) := x^2 - 1$ means that the function f is defined to be $x^2 - 1$, or $A := \{1, 5, 7\}$ means that the set A is defined to be $\{1, 5.7\}$.
$S_1 \Rightarrow S_2$	*implies*	Logical implication: If statement S_1 is true, then statement S_2 must be true. We say S_1 *is a sufficient condition for* S_2 or S_2 *is a necessary condition for* S_1.
		Examples: $(n \in \mathbb{N}$ even$) \Rightarrow (n^2$ even$)$.
$S_1 \Leftrightarrow S_2$	*equivalent to*	Logical equivalence: If statement S_1 is true, then statement S_2 must be true, and vice versa. We say S_2 *is a necessary and sufficient condition for* S_1.
		Examples: $(\ln x > 0) \Leftrightarrow (x > 1)$.
$\exists$	*there exists*	Abbreviation for *there exists*
$\forall$	*for all*	Abbreviation for *for all*
$\{\dots\}$	*set*	The "objects" listed between the curly brackets are members of the set being defined.
		Examples: $\{0, 2, 5, 7\}$, $\{2 + i, 7 - \sqrt{5}\}$, $\{$, , $\}$ The elements of a set can be any kind of objects such as numbers, functions, points, geometric objects or other.
$a \in A$	*element of*	a is an element of the set A, that is, a is in the set A.
		Examples: $\pi \in \mathbb{R}$, $4 \in \{1, 4, 7\}$, $\in \{$, , $\}$
$\emptyset$ or $\{\}$	*empty set*	The special set that does not contain any element.
$\{x \mid \text{property}\}$	*set of ... with ...*	Notation indicating a set of elements x satisfying a certain property.
		Examples: $\{n \in \mathbb{N} \mid n \text{ is even}\}$, where $n \in \mathbb{N}$ is the typical element and the property satisfied is that n is even. $\{x^2 \mid x \in \mathbb{N}\}$, where the typical member is a square of some number in $\mathbb{N}$.
$A \subseteq B$	*subset of*	The set A is a subset of B, that is, every element of A is also an element of B. More formally: $b \in B \Rightarrow b \in A$.
		Examples: $\mathbb{Q} \subseteq \mathbb{R}$, $\{1, 4, 7\} \subseteq \{1, 2, 3, 4, 5, 6, 7\}$
$A \cup B$	*union*	The set of elements either in A or in B. More formally: $(x \in A \cup B) \Leftrightarrow (x \in A$ or $x \in B)$.
		Examples: $\{1, 4, 7\} \cup \{4, 5, 8\} = \{1, 4, 5, 7, 8\}$ (elements are not repeated in a union if they appear in both sets!)
		Note: We can look at a union of an arbitrary collection of sets: The set of objects that appear in at least one of the sets in the collection.
$A \cap B$	*intersection*	The set of elements that are in A and in B. More formally: $(x \in A \cap B) \Leftrightarrow (x \in A$ and $x \in B)$.
		Examples: $\{1, 4, 7\} \cap \{1, 2, 3, 5, 6, 7\} = \{1, 7\}$
		Note: We can look at the intersection of an arbitrary collection of sets: The set of objects that appear in every set in the collection.
$A \setminus B$	*complement*	The set of elements that are in A but not in B. More formally: $(x \in A \setminus B) \Leftrightarrow (x \in A$ and $x \notin B)$.
		Examples: $\{1, 4, 5, 7\} \setminus \{1, 2, 3, 6, 7\} = \{4, 5\}$

Interval notation

Notation	Terminology	Explanation and Examples
$[a, b]$	*closed interval*	If $a, b \in \mathbb{R}$ with $a \leq b$ the closed interval is the set $\{x \in \mathbb{R} \mid a \leq x \leq b\}$
		Examples: $[-3, 5]$ is the set of real numbers between -3 and 5, including the endpoints -3 and 5.
(a, b)	*open interval*	If $a, b \in \mathbb{R}$ with $a \leq b$ the open interval is the set $\{x \in \mathbb{R} \mid a < x < b\}$
		Examples: $(-3, 5)$ id the set of real numbers between -3 and 5, excluding the endpoints -3 and 5.
$[a, b)$ or $(a, b]$	*half open interval*	If $a, b \in \mathbb{R}$ with $a \leq b$, $[a, b)$ is the set of all numbers between a and b with a included and b excluded. In case of $(a, b]$ the endpoint a is excluded and b is included.
		Examples: $[-3, 5)$ is the set of real numbers between -3 and 5, including -3 but excluding 5. For $(-3, 5]$ the endpoint -3 is excluded and 5 is included.
$[a, \infty)$ or $(-\infty, a]$	*closed half line*	If $a \in \mathbb{R}$, then $[a, \infty)$ is the set of real numbers larger than or equal to a, and $(-\infty, a]$ is the set of real numbers less than or equal to a
(a, ∞) or $(-\infty, a)$	*open half line*	If $a \in \mathbb{R}$, then (a, ∞) is the set of real numbers strictly larger than a, and $(-\infty, a)$ is the set of real numbers strictly less than a
		Examples: $(0, \infty)$ set of all positive real numbers; $(-\infty, 5]$ set of all real numbers less than or equal to 5.

Functions

Notation	Terminology	Explanation and Examples
$f : A \to B$	*function*	A function f from the set A to the set B is a rule that assigns every element $x \in A$ a unique element $f(x) \in B$. The set A is called the *domain* and represents all possible (or desirable) "inputs", the set B is called the *codomain* and contains all potential "outputs".
$x \mapsto f(x)$	*is mapped to*	The function maps x to the value $f(x)$.
		Examples: $g : \mathbb{R} \to \mathbb{C}, \theta \mapsto g(\theta) := e^{i\theta}$. A function from $\mathbb{R}$ to $\mathbb{C}$ given by $e^{i\theta}$; $f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) := 1 + x^2$. A function from $\mathbb{R}$ to $\mathbb{R}$ given by $1 + x^2$; $h : \mathbb{C} \to [0, \infty), z \mapsto h(z) := \lvert z \rvert$. A function from $\mathbb{C}$ to $[0, \infty)$ given by $\lvert z \rvert$.
$\mathrm{im}(f)$	*image or range*	The set of values $f : A \to B$ attains: $\mathrm{im}(f) := \{f(x) : x \in A\} \subseteq B$.
		Examples: $f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) := x^2$. The codomain is $\mathbb{R}$, the image or range is $[0, \infty)$.
	surjective or onto	A function $f : A \to B$ is called *surjective* if $\mathrm{im}(f) = B$, that is, the codomain coincides with the range. More formally: For every $b \in B$ there exists $a \in A$ such that $f(a) = b$.
		Note: $f : A \to \mathrm{im}(f)$ is always surjective. The choice of codomain is quite arbitrary. We often just state the general objects rather than the image or range. For instance function values are in $\mathbb{R}$ if we are not intested in the image.
	injective or one-to-one	A function $f : A \to B$ is called *injective* if $\mathrm{im}(f) = B$, that is, every point in the image comes from exactly one point in the domain A. More formally: If $a_1, a_2 \in A$ are such that $f(a_1) = f(a_2)$, then $a_1 = a_2$.
	bijective	A function $f : A \to B$ is called *bijective* if it is surjective and injective.
f^{-1}	*inverse function*	A function $f : A \to B$ is called *invertible* if it is bijective. The inverse function $f^{-1} : B \to A$ is defined as follows: Given $b \in B$ take the unique point $a \in A$ such that $f(a) = b$ and set $f^{-1}(b) := a$ (by surjectivity such a exists, by injectivity it is unique).